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NUMERICAL STUDY OF SELF-INDUCED WING ROCK OF A LOW-ASPECT-RATIO DELTA WING IN THE PRESENCE OF EXTERNAL DISTURBANCES

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Self-induced wing rock of a delta wing, in particular, in the presence of external disturbances are studied by means of numerical simulations of a separated flow of an ideal incompressible fluid around a delta wing. The results obtained are compared with experimental data. The vortex nature and the mechanism of self-induced oscillations are studied. Regions of synchronization of the aerodynamic self-oscillatory system in the presence of external disturbances are identified. Methods of suppression of self-induced wing rock are proposed.

Key words: *numerical study, delta wing, self-induced wing rock, external disturbances, capture of frequency, suppression of self-induced oscillations.*

Separated flows at high angles of attack can include undesirable self-induced roll-oscillations of flying vehicles. It is known that such oscillations can precede the regime of the flying vehicle going into a spin. In this case, there occur self-induced oscillations owing to flow separation from the leading edges of the lifting surfaces. It seems reasonable to study this phenomenon by an example of simple lifting surfaces, such as delta wings. Self-induced oscillations of such wings were studied experimentally in [1–4], and numerical simulations of this phenomenon were performed in [5–8]. The present paper reports the results of a numerical study of self-induced wing rock of a delta wing, in particular, in the presence of external disturbances.

1. A mathematical model of unsteady motion of a slender delta wing in an ideal incompressible fluid is considered. The following assumptions of the nonlinear theory of the wing in a separated ideal fluid flow are used: the flow outside the wing and the vortex wake is potential; the separation line on the sharp edges is fixed; there are no secondary separations; the no-slip conditions on the wing, the Joukowski postulate on the edges, the kinematic and dynamic conditions on the vortex sheet, and the condition of decaying disturbances at infinity are satisfied.

The wing motion is assumed to start moving from the state at rest with a constant velocity V. At the time t_0, the wing with a stable vortex structure formed above acquires a degree of freedom in terms of rolling. The rotation axis is aligned along the root chord of the wing (Fig. 1). Beginning from the time t_0, the equations of unsteady aerodynamics are solved together with the equation of wing motion in terms of its rolling

$$\ddot{\gamma} = c_1 m_x(\gamma, \dot{\gamma}, \alpha, \lambda) \tag{1}$$

with the initial conditions (initial deviations in terms of rolling at the time τ_0)

$$\gamma(\tau_0) = \gamma_0, \qquad \dot{\gamma}(\tau_0) = 0. \tag{2}$$

In Eqs. (1) and (2), γ is the rolling angle, α is the angle of attack, $\tau = tV/b$ is the dimensionless time, t is the time, b is the length of the root chord, λ is the aspect ratio, $m_x = 2M_x/(\rho V^2 Sl)$, $c_1 = \rho b^2 Sl/(2I_x)$, M_x is the rolling moment, ρ is the fluid density, and I_x, S, and l are the moment of inertia, the wing area, and the wing span, respectively.

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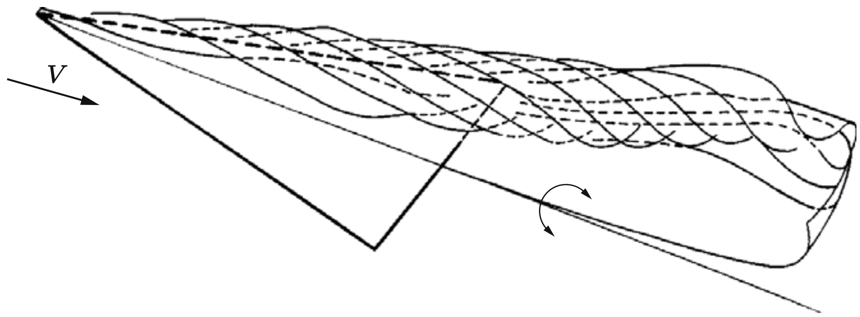

Fig. 1. Vortex structure above the wing starting its motion from the state at rest.

The algorithm of the numerical solution of the nonlinear problem of the separated unsteady flow around a delta wing is constructed on the basis of the method of discrete vortices [5–8]. A rectangular vortex cascade (the same as that in [7]) was used in the calculations. The number of vortices along the chord and over the wing span was doubled: 12 and 24, respectively.

2. The numerical results obtained were compared with experimental data. The values of the physical parameters in the calculations corresponded to those obtained in the experiments [3]. With allowance for the moment of inertia of a real wing with an aspect ratio $\lambda = 0.71$ ($I_x = 0.00039$ kg $\cdot$ m $\cdot$ sec^2), the coefficient in Eq. (1) took the value $c_1 = 4.35$. The experiment was performed on a delta wing with a symmetric sharp edge whose relative thickness was $\bar{c} = 0.025$ (the thickness was normalized to the wing chord b).

Figure 2 shows the calculated and experimental dependences of the amplitude A and dimensionless eigenfrequency ω_0 [$\omega_0 = \omega l/(2V)$, where ω is the circular frequency] on the angle of attack α in the case of self-induced oscillations of the wing. It is seen that the numerical and experimental results are in reasonable agreement.

3. Some results, which allow the wing rock mechanism for the delta wing to be clarified, are given below. It follows from the analysis of the experimental data [1–3] that self-induced wing rock does not appear if the angle of attack does not exceed a certain critical angle α_*. If $\alpha > \alpha_*$, then the self-induced wing rock can be caused by unsteadiness of the fluid or by a non-zero initial rolling angle of the wing specified in the calculations.

Figure 3 shows two integral curves of Eq. (1) on the phase plane $(\gamma, \dot{\gamma})$. The following initial conditions (2) were imposed: $\gamma(\tau_0) = 20°$ and $\dot{\gamma}(\tau_0) = 0$. For $\alpha = 15°$ ($\alpha < \alpha_*$), the integral curve has the form of a spiral rolling inward. In this case, the origin is a singular point of the stable focus type, and the wing oscillations decay. At $\alpha = 27°$ ($\alpha > \alpha_*$), the integral curve becomes rolled up from inside onto the limiting cyclic curve, and self-induced oscillations arise in the system.

It should be noted that damping prevails in the system in the first half-period of wing motion ($\alpha > \alpha_*$), which begins at a certain constant rolling angle. It is manifested as a decrease in the deviation of the rolling angle from the origin: $\gamma_1 > \gamma_2$ (see Fig. 3). Nevertheless, anti-damping prevails already in the next half-period: the deviation increases and the integral curve tends to the limiting cyclic curve. Thus, anti-damping does not arise in the system immediately; it arises only in the second half-period from the moment of the emergence of free oscillations of the wing.

The mechanism of the emergence of anti-damping, which has an unsteady vortex nature, in the oscillatory system considered was described in [7] on the basis of studying the vortex structures on the wing. It is shown that the right half of the wing (panel) moves away from the above-located vortex cord if the wing moves clockwise in terms of rolling, whereas the left panel approaches the corresponding vortex cord. Because of the delay of motion of the vortex cords behind the moving wing, however, the left vortex cord is closer than the right vortex cord to the corresponding panel in the vicinity of the neutral position of the wing ($\gamma = 0$). This asymmetry of the vortex cords induces a moment in the direction of wing rotation.

The further motion of the wing is accompanied by a decrease in its angular velocity and, hence, by an increase in time necessary for reconstruction of the vortex cords to positions corresponding to the steady flow regime. Moreover, the leeward vortex cord is shifted outside the wing, and damping appears again.

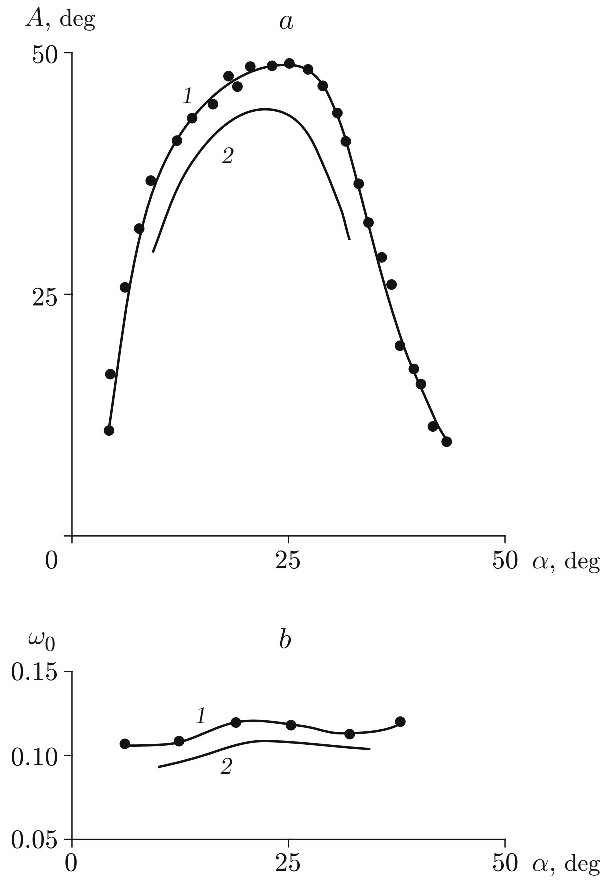

Fig. 2. Calculated and experimental dependences of the amplitude A (a) and dimensionless eigenfrequency ω_0 (b) versus the angle of attack in the case of self-induced roll-oscillations of the wing ($\lambda = 0.71$): experimental data [3] (1) and calculations performed in the present work (2).

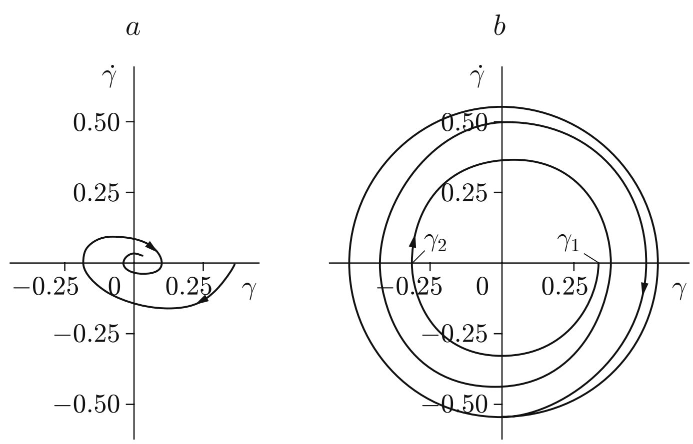

Fig. 3. Integral curves of wing oscillations for different angles of attack: (a) $\alpha = 15^\circ$ (decay of oscillations); (b) $\alpha = 27^\circ$ (oscillations reach a limiting cycle).

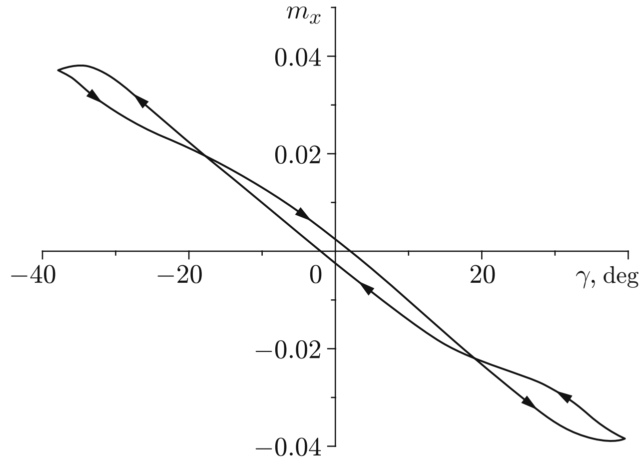

Fig. 4. Dependence $m_x(\gamma)$ in the case of the wing rock.

Thus, anti-damping (and, finally, self-induced oscillations) in the vicinity of the neutral position of the wing is caused by unsteadiness of the vortex process of the flow around the wing.

It should be noted that the above-mentioned delay in motion of the vortex cords occurs only in the second half-period from the moment of the emergence of free oscillations of the wing; as a result, anti-damping prevails in the system until it reaches the limiting cycle. In this case, damping and anti-damping in the oscillatory system are balanced over the period.

This regime of the aerodynamic self-oscillatory system is described by the loop $m_x(\gamma)$ in Fig. 4; the arrows indicate the path direction during the period of oscillations. It is seen that the loop has internal intersections at two points, which is confirmed by the experimental data [3]. The presence of closed parts of the loop $m_x(\gamma)$ in the anticlockwise path testifies to damping on these areas in the system. The part of the loop with the clockwise path in the vicinity of the neutral position of the wing ($\gamma = 0$) corresponds to the time interval with prevailing of anti-damping in the system. The areas of these regions with different directions of the path are almost identical, because the system is in the limiting cycle of oscillations in this case.

4. During the flight, the lifting elements of the wing can be affected by external periodic disturbances caused, in particular, by incoming flow unsteadiness. Let us study the influence of such external disturbances on the aerodynamic self-oscillatory system of the delta wing.

According to the theory of nonlinear oscillations [9], "capture" of frequency is possible when a self-oscillatory system is subjected to external disturbances. If this "capture" occurs in the vicinity of the system eigenfrequency ω_0, i.e., in the vicinity of the frequency of self-induced oscillations of the wing in the absence of external disturbances, it is called harmonic. If the "capture" occurs at a frequency whose value is greater or smaller than the system eigenfrequency by an integer factor, it is called ultraharmonic or subharmonic, respectively.

Let an external disturbance determined by a harmonic law be applied beginning from a certain time to the aerodynamic self-oscillatory system. In this case, the equation of the wing motion in terms of rolling takes the form

$$\ddot{\gamma} = c_1(m_x + m_{xv}) = c_1[m_x + m_{xv0}\sin(\omega_v\tau + \varphi_v)].$$

Here, m_{xv} is the dimensionless coefficient of the rolling moment of the external disturbance, ω_v and φ_v are the dimensionless frequency and phase shift of the external disturbance, and m_{xv0} is the dimensionless amplitude of the moment of the external disturbance.

We consider a delta wing with an aspect ratio $\lambda = 0.71$ aligned at an angle of attack $\alpha = 25°$ exceeding the angle α_* at which self-induced oscillations arise. It was shown [7] that a decrease in the moment of inertia of the wing increases the frequency of self-induced oscillations and reduces the time necessary for the system to reach a steady regime. Therefore, the study was performed with a wing whose moment of inertia was smaller than that of the wing considered in Sec. 2.

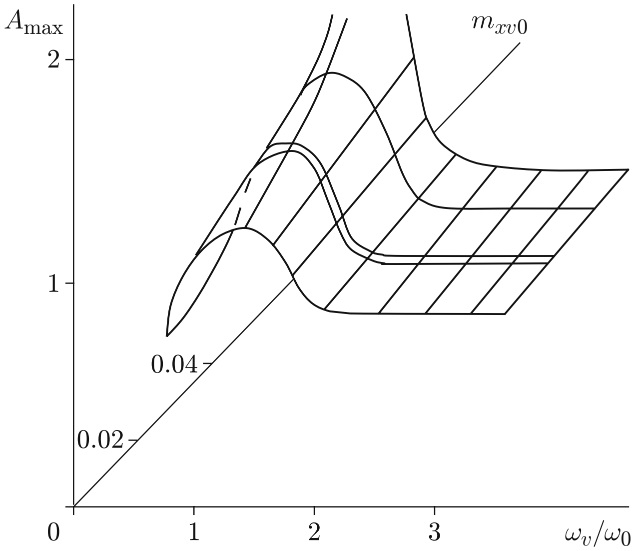

Fig. 5

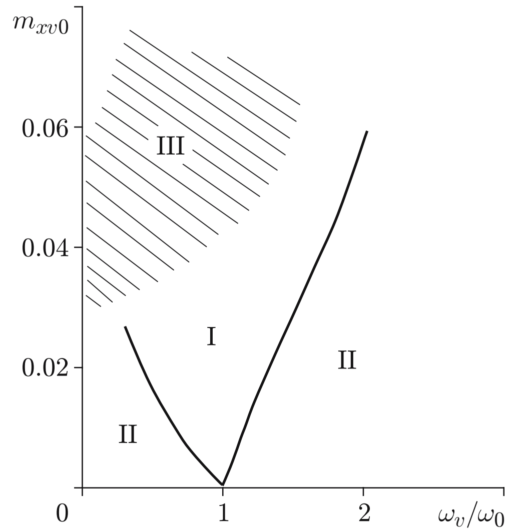

Fig. 6

Fig. 5. Maximum amplitude of self-induced oscillations of the wing $A_{\max}$ versus the frequency and amplitude of the external disturbance ($\alpha = 25°$ and $\lambda = 0.71$).

Fig. 6. Regions of synchronization of self-induced oscillations of the wing ($\alpha = 25°$ and $\lambda = 0.71$): region of the harmonic "capture" of frequency (I), regions where the eigenfrequency of the self-oscillatory system prevails (II), and region where the oscillations disappear and wing rotation about the root chord begins (III).

The dimensionless eigenfrequency of self-induced oscillations of the considered wing was $\omega_0 = 0.178$. The parameters varied in numerical calculations were the amplitude and frequency of the external disturbance. Using the results of a series of conducted calculations, we constructed the dependence of the maximum amplitude $A_{\max}$ of self-induced oscillations on the ratio of the dimensionless frequencies ω_v/ω_0 and the external disturbance amplitude m_{xv0} (Fig. 5). Let us consider the cross section of this surface by the plane $m_{xv0} = 0.02$, which is shown in Fig. 5 by the double curve. It is seen that the value of $A_{\max}$ increases along this curve as $\omega_v/\omega_0 \to 1$ (when the frequency of external disturbances is in the vicinity of the eigenfrequency of self-induced oscillations). The maximum value of the amplitude of oscillations $A_{\max}$ is reached at $\omega_v \approx \omega_0$.

In other sections $m_{xv0} = \text{const} > 0.02$, the value of $A_{\max}$ increases with increasing external disturbance amplitude m_{xv0}, but this increase is not infinite. Thus, at $m_{xv0} > 0.04$, self-induced oscillations of the delta wing in terms of rolling cease to exist, and the wing starts rotating in one direction with respect to the root chord.

It should be noted that $A_{\max}$ takes a constant value, which is equal to the amplitude of self-induced oscillations of the delta wing in the absence of external disturbances, for all considered values of the parameter m_{xv0} at large and small values of the dimensionless frequency of forced oscillations ω_v (outside the vicinity $\omega_v \approx \omega_0$). Thus, the influence of the external disturbance on the amplitude of self-induced oscillations of the delta wing is actually manifested only in the vicinity of the eigenfrequency of the system.

To determine the regions of synchronization of the aerodynamic self-oscillatory system, we performed a harmonic analysis of the dependences of the rolling angle on the dimensionless time $\gamma(\tau)$. Based on the calculations performed, three regions of synchronization were found in the plane $(\omega_v/\omega_0, m_{xv0})$ (Fig. 6). Region I of the harmonic "capture" of frequency of the aerodynamic self-oscillatory system is similar to the corresponding region obtained with the use of the van der Paul equation [9]. In contrast to the van der Paul oscillator, however, the transition from wing oscillations to its rotation is observed if the external disturbance amplitude m_{xv0} is sufficiently high (region III in Fig. 6). An apparent reason is the nonlinearity of the restoring force manifested at high angles of rolling, which leads to a change in the wing motion equation type. The eigenfrequency of the self-oscillatory system prevails in region II.

5. Some methods of suppression of self-induced roll-oscillations of the delta wing (without using automatic control systems) developed with the use of numerical calculations are given below:

— change in the angle of attack of the wing to values that do not allow self-induced oscillations ($\alpha < \alpha_*$);

— choice of a negative value of the angle of the transverse V-wing;

— upward deflection of individual fragments of the leading edge of the wing;

— increase in the moment of inertia of the wing.

Thus, the study performed provided more details of one type of self-oscillatory regimes of the flow around the lifting surfaces of flying vehicles, in particular, with allowance for external disturbances.

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